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E. g. Take $p=2$ and $q=1$. Then $x=30$, $2a=66$, $b=55$, and $c=44$.

Of course, we can take $a=2pq$, and $c=p^2-q^2$; but the resulting expression is not so simple and yet when reduced to integrals gives the same value for x, but cone thus obtained has different dimensions.

Also solved by G. B. M. Zerr, who gets slant height$=605m^2$, altitude$=484m^2$, radius of sphere $=330m^2$, and radius of cone$=363m^2$, where m is any integer.

98. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford University, Cal.

(*a*) Find the least three integral numbers such that if to the square of each the product of the other two be added, the three sums shall all be squares.

(*b*) Find the two least integral numbers such that not only each of them, but also their sum and their difference, when increased by unity, shall all be square numbers.

Solution by HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.

(*a*) Let x^2, m^2x^2 and nx^2 represent the required numbers. Then we have $1+mn=\square$....(1); $m^2+n=\square$....(2), and $m+n^2=\square$....(3). Take $n=\frac{1}{4}-m$ ....(4), and (2) becomes a square, as (3) does, when we transpose (4) and get $m=\frac{1}{4}-n$. Substitute the value of n in (1) and we have

$$1+\frac{m}{4}-m^2=\square=(\text{say})\ [1-pm]^2.$$

Reducing, we have $m=\dfrac{8p+1}{4[p^2+1]}$ and $n=\frac{1}{4}-m=\dfrac{p[p-8]}{4[p^2+1]}$.

Take $x=4[p^2+1]$, $mx=8p+1$, and $nx=p[p-8]$, in which p may be any number greater than 8. Take $p=9$ and we have $x=328$, $mx=73$, and $nx=9$.

(*b*) Let x^2 and y^2 be the numbers. Then $x^2+y^2+1=\square=(\text{say})(m+1)^2$ and $x^2-y^2+1=\square=(\text{say})(m-1)^2$.

By adding and reducing, we get $x^2=m^2$ and $x=m$.

By substituting and reducing, we find $y^2=2m$. Take $m=2n^2$ and we have $x=2n^2$ and $y=2n$. Take $n=1$, and $x=2$, and $y=2$, and the numbers are 4 and 4; take $n=2$ and the numbers are 16 and 64.

Also solved by *G. B. M. ZERR*, and *L. C. WALKER*.

AVERAGE AND PROBABILITY.

Remark on Problem 109, by G. B. M. ZERR, A. M., Ph. D.. The Temple College. Philadelphia. Pa.

Let $r=a\sin^2\theta$. Then the last integral in Professor Walker's solution is transformed as follows:

$$\frac{8}{9a}\int_0^a[3a^3\tan^{-1}(\frac{r}{a-r})-(3a^2+2ar-8r^2)\surd(ar-r^2)]dr,$$

$$=\frac{16a^3}{9}\int_0^{\frac{1}{2}\pi}(3\theta-3\sin\theta\cos\theta-2\sin^3\theta\cos\theta+8\sin^5\theta\cos\theta)\sin\theta\cos\theta d\theta,$$

$$=\frac{16a^3}{9}\int_0^{\frac{1}{2}\pi}(3\theta\sin\theta\cos\theta-3\sin^2\theta+\sin^4\theta+10\sin^6\theta-8\sin^8\theta)d\theta,$$

$$=\frac{16a^3}{9}(\tfrac{3}{8}\pi-\tfrac{3}{4}\pi+\tfrac{3}{16}\pi+\tfrac{25}{16}\pi-\tfrac{35}{32}\pi)=\frac{16a^3}{9}\times\frac{9\pi}{32}=\tfrac{1}{2}\pi a^3.$$

117. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics. The Temple College, Philadelphia, Pa.

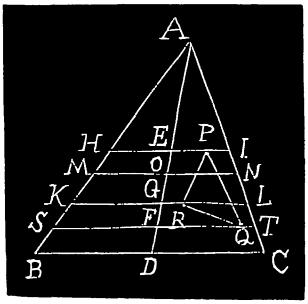
A straight line is drawn at random parallel to the base of a given triangle. Three random points are then taken, one on each side of the random line and one anywhere in the triangle. Find the average area of the triangle formed by the three random points.

Solution by the PROPOSER.

Let ABC be the given triangle, MN the random line parallel to the base BC. Take P above, Q below MN, and R anywhere. Through P, Q, R draw HI, ST, KL parallel, respectively, to MN. Let $BC=2a$, $BD=DC=a$, $AD=h$, $\angle ADC=\beta$, $AE=u$, $AF=v$, $AG=w$, $EP=x$, $GR=z$, $FQ=y$, $AO=r$.

Then $EI=\dfrac{au}{h}=x'$, $GL=\dfrac{aw}{h}=z'$, $FT=\dfrac{av}{h}=y'$, $GJ=y-\dfrac{(y-x)(v-w)}{v-u}=t$.

Area $PQR=\frac{1}{2}(t-z)(v-u)\sin\beta=A$, $t>z$; area $PQR=\frac{1}{2}(z-t)(v-u)\sin\beta=A_1$, $t<z$. The limits of r are 0 and h; of v, r and h; of u, 0 and r; of w, u and v; of x, $-x'$ and x'; of y, $-y'$ and y'; of z, $-z'$ and t, and t and z'.



$$\therefore\triangle=\frac{\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[\int_{-z'}^{t}Adz+\int_t^{z'}A_1dz\right]drdvdydudxdw}{\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\int_{-z'}^{z'}drdvdydudxdwdz}$$

$$=\frac{15}{a^3h^4}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[\int_{-z'}^{t}Adz+\int_t^{z'}A_1dz\right]drdvdydudxdw$$

$$=\frac{15\sin\beta}{2a^3h^6}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[h^2t^2+a^2w^2\right](v-u)drdvdydudxdw$$

$$=\frac{5\sin\beta}{2a^3h^6}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\left[a^2(u^2+uv+v^2)+h^2(x^2+xy+y^2)\right]$$

$$\times(v-u)^2drdvdydudx$$